Lecture Notes, Econ 320B. Set # 2.

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1 Excess Demand, Walrasian Equilibrium

Last week we looked at a consumer $i \in \mathscr{I}$'s *Utility Maximization Problem*:

$$\begin{aligned} &\max \ u^i(x_1^i, \ldots, x_n^i) \\ &\text{s.t.} \begin{cases} \sum_k p_k x_k^i \leq \sum_k p_k e_k^i \\ x_k^i \geq 0 \text{ for } k = 1, \ldots, n \end{cases} \end{aligned} \tag{1}$$

The central assumption is:

Assumption 5.1. (JR p.188) *The utility function $u^i : \mathbb{R}_+^n \to \mathbb{R}$ is continuous, strongly increasing, and strictly quasi-concave.*

Under Assumption 5.1. we got Theorem 5.1. (JR p.189) which basically says that there'll exist a demand function for consumer i, $\mathbf{x}^i(\mathbf{p}, \mathbf{p}\mathbf{e}^i)$, which will be continuous in $\mathbf{p}$ for all $\mathbf{p} \gg 0$.

The k'th coordinate of $\mathbf{x}^i(\mathbf{p}, \mathbf{p}\mathbf{e}^i)$ (the demand for the k'th good) is written $x_k^i(\mathbf{p}, \mathbf{p}\mathbf{e}^i)$. We now define:

Definition 5.4. (JR p.189) *The aggregate excess demand function for good $k \in \{1, \ldots, n\}$ is the real-valued function:*

$$z_k(\mathbf{p}) = \sum_{i \in \mathscr{I}} x_k^i(\mathbf{p}, \mathbf{p}\mathbf{e}^i) - \sum_{i \in \mathscr{I}} e_k^i$$

The aggregate excess demand function is the vector-valued function:

$$\mathbf{z}(\mathbf{p}) = (z_1(\mathbf{p}), \ldots, z_n(\mathbf{p}))$$

A **Walrasian Equilibrium** (often written simply **WE**), is a price vector $\mathbf{p}^* \in \mathbb{R}_{++}^n$ such that "aggregate demand" $\sum_i \mathbf{x}^i(\mathbf{p}^*, \mathbf{p}^*\mathbf{e}^i)$ equals "aggregate supply" $\sum_i \mathbf{e}^i$. That is to say:

$$\sum_i \mathbf{x}^i(\mathbf{p}^*, \mathbf{p}^*\mathbf{e}^i) = \sum_i \mathbf{e}^i$$

Or written in terms of the n goods:

$$\sum_i x_k^i(\mathbf{p}^*, \mathbf{p}^*\mathbf{e}^i) = \sum_i e_k^i \text{ , for } k = 1, \ldots, n$$

It is clear that $\mathbf{p}^*$ is a WE if and only if $\mathbf{z}(\mathbf{p}^*) = 0$, that is to say, if and only if aggregate excess demand equals zero (or, more accurately, equals the zero vector). This is Definition 5.5. in the book.

When $\mathbf{p}^*$ is a WE, the associated allocation is called a **Walrasian Equilibrium Allocation** (or simply **WEA** for short). You'll find the definition on p.199 in GR:

Definition 5.6. Let $\mathbf{p}^*$ be a WE for an economy with initial endowments $\mathbf{e}$. Then $\mathbf{x}(\mathbf{p}^*)$ defined as follows is called a **Walrasian Equilibrium Allocation** (**WEA**):

$$\mathbf{x}(\mathbf{p}^*) = (\mathbf{x}^1(\mathbf{p}^*, \mathbf{p}^*\mathbf{e}^i), \ldots, \mathbf{x}^I(\mathbf{p}^*, \mathbf{p}^*\mathbf{e}^i))$$

Of course the WEA associated with a given WE is nothing but the "resulting demands for the I consumers at the equilibrium prices $\mathbf{p}^*$".

2 Basic Properties of Excess Demand Functions

At the lectures we are going to prove the following important result (at least we're going to prove parts 2. and 3., we won't speak a lot about continuity though the claim is actually trivial if you know just a little bit about continuity):

Theorem 5.2.* **(Properties of Excess Demand Functions)** *If for each consumer* i, u^i *satisfies assumption 5.1., then for all* $\mathbf{p} \gg 0$.

1. *Continuity:* $\mathbf{z}(\mathbf{p})$ *will be continuous in* $\mathbf{p}$.
2. *Homogeneity:* $\mathbf{z}(\lambda\mathbf{p}) = \mathbf{z}(\mathbf{p})$ *for all* $\lambda > 0$
3. *Walras' law:* $\mathbf{p}\mathbf{z}(\mathbf{p}) = 0$.

That the Theorem has a little "star" (*) attached to it means that you may be asked to prove this result at the exam (or midterm). So you need to understand and remember this proof. Fortunately, it isn't all that difficult. Since we do not speak about continuity, you will not be asked to prove part 1.

3 Existence of Walrasian Equilibrium

It is important to know whether a WE actually exists for any given economy. At the lecture I'll speak a bit about why this matters a great deal, also from a "socio-political" perspective. At any rate, the key result on exchange economies is the following from p.196 in JR:

Theorem 5.5. (Existence of WE) If each consumer's utility function satisfies assumption 5.1. and $\sum_{i\in\mathcal{I}} \mathbf{e}^i \gg 0$, then there exists at least one WE, *i.e.*, a price vector $\mathbf{p}^* \gg 0$ such that $\mathbf{z}(\mathbf{p}^*) = 0$.

Notice here that the theorem says "at least". There may be more than one, so if we let an exchange economy run its course and it (hopefully) ends up in an equilibrium, we cannot always say precisely where it is going to end up. This is a little depressing in a sense, but on the other hand it leads to some interesting additional questions such as: If there are multiple WEs (and associated WEAs), can we through economic policy somehow *steer* the economy to one which we "like" better ? We'll talk more about this later, including the word "like" (what does it mean to like something, in particular, can a bunch of people always agree on what they like the most?).