Lecture Notes, Econ 320B. Set # 5.

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1 Feasible Allocations, Pareto Optimality, the First Welfare Theorem

1.1 Feasibility

Remember from last week's notes that a **Walrasian equilibrium** in a private ownership economy with production $(u^i, e^i, \theta^{ij}, Y^j)_{i\in\mathscr{I}, j\in\mathscr{J}}$ is a price vector $p^* \gg 0$ such that the markets clear:

$$\sum_{i\in\mathscr{I}} x^i(p^*, m^i(p^*)) = \sum_{i\in\mathscr{I}} e^i + \sum_{j\in\mathscr{J}} y^j(p^*)$$

It is clear that the associated WEA is **feasible** in the sense of the following definition [this definition is found on p.217 in GR. It is given "in-text", i.e., it doesn't get a formal definition environment. Nonetheless, it is of course a definition].

Definition 1 **(Feasible Allocations)** *An allocation* $(\mathbf{x}, \mathbf{y}) = ((\mathbf{x}^1, \ldots, \mathbf{x}^I), (\mathbf{y}^1, \ldots, \mathbf{y}^J))$ *of bundles to consumers and production plans to firms is* **feasible** *if:*

1. $\mathbf{x}^i \in \mathbb{R}^n_+$ *for all* $i \in \mathscr{I}$ *[Consumption plans lie in consumption sets]*
2. $\mathbf{y}^j \in Y^j$ *for all* $j \in \mathscr{J}$ *[Production plans lie in production sets]*
3. $\sum_{i\in\mathscr{I}} \mathbf{x}^i = \sum_{i\in\mathscr{I}} \mathbf{e}^i + \sum_{j\in\mathscr{J}} \mathbf{y}^j$ *[Markets clear]*

1.2 Pareto Optimality

As in the exchange economy case we can now define Pareto optimality. This is definition 5.9 (page 217) in GR.

Definition 2 **(Pareto Optimality)** *A feasible allocation* $(\mathbf{x}, \mathbf{y})$ *is* **Pareto optimal** *(or* **Pareto efficient***) if there does not exist another feasible allocation* $(\tilde{\mathbf{x}}, \tilde{\mathbf{y}})$ *such that* $u^i(\tilde{\mathbf{x}}^i) \geq u^i(\mathbf{x}^i)$ *for all* $i \in \mathscr{I}$ *with at least one strict inequality.*

Note that firms "enter" the definition of Pareto optimality through feasibility: For an allocation to be feasible, production plans must be technologically feasible (lie in the production sets). Thus, when the Pareto optimal allocation in the definition is compared against other *feasible* allocations, we are only comparing against other allocations which the economy's production capabilities can actually "sustain".

1.3 The First Welfare Theorem with Production

As in exchange economies, free markets (perfect competition/price taking behavior) leads to Pareto optimality. This is Theorem 5.14 in GR. Notice that the assumption in the theorem is the exact same as the one in exchange economies.

Theorem 1 * **(First Welfare Theorem with Production)** *If each u^i is strictly increasing on $\mathbb{R}^n_+$, then every WEA is Pareto optimal.*

We shall prove this result in class :-)

The first welfare theorem with production is a very strong result in the sense that it requires almost no assumptions. Basically, we can say that as long as (i) consumers are greedy (each u^i strictly increasing), and (ii) everyone takes prices as given, *then* we get Pareto optimality. It is not important for this result that, say, u^i is strictly quasi-concave, Y^j strongly convex, etc. Of course such assumptions may be necessary in order to prove existence but it is important that you don't confuse these issues: As long as a WE exists, the associated WEA will be Pareto optimal when each u^i is strictly increasing. A WE could exist under lots of different conditions, in last week's notes we saw an existence result requiring certain conditions, but other sets of conditions would lead to existence too - and in either case the first welfare theorem will apply.

2 Private Ownership Economies with (Lump-sum) Transfers

2.1 Transfers, Redistributions

Before we looked at private ownership economies (without transfers) where the income of a consumer $i \in \mathscr{I}$ is given by:

$$m^i(p) = pe^i + \sum_{j=1}^{J} \theta^{ij} \Pi^j(p)$$

Now we introduce (lump-sum) income transfers. Specifically, **(a sequence of lump-sum) income transfers** is a vector $(T_1, \ldots, T_I) \in \mathbb{R}^I$. Given such a sequence the income of consumer i is taken to be:

$$m^i(p) = pe^i + \sum_{j=1}^{J} \theta^{ij} \Pi^j(p) + T_i$$

Notice that T_i may be positive, negative, or zero. If it is positive $T_i > 0$, the consumer is a *recipient* of income. If $T_i < 0$, the consumer is a *payee* (she pays income in the form of taxes). And if $T_i = 0$, she is not affected by the transfers. When

$$\sum_{i \in \mathscr{I}} T_i = 0$$

we call the (sequence of) income transfers is a **redistribution**. A redistribution is exactly what the name says: A *re*-distribution of income among consumers, in particular there is no government that grabs any income in the aggregate.

Because of the way these transfers enter into the income equation (they are simply "added" before-transfers incomes), these are called **lump-sum** transfers. Intuitively, consumers wake up in the morning and the amount T_i has been debited or credited to their bank account depending on whether it is positive or negative. This is in sharp contrast with other kinds of taxes such as VAT and income taxes which place a "wedge" between buyers' and sellers' prices (think of income taxes: the firm pays X per unit of labor, the wage, and the worker only gets for example $(1-t)X$, per unit where t is the rate of taxation, here proportional).

2.2 Equilibrium in private ownership economies with transfers

It is relatively obvious how we go about defining a **Walrasian equilibrium** in a private ownership economy ($\sum_i \theta^{ij} = 1$ all j) with income transfers: As a price vector $\mathbf{p}^* \gg 0$ such that markets clear,

$$\sum_{i \in \mathscr{I}} x^i(p^*, m^i(p^*)) = \sum_{i \in \mathscr{I}} e^i + \sum_{j \in \mathscr{J}} y^j(p^*)$$

This is exactly as before, except that now $m^i(p) = pe^i + \sum_{j=1}^{J} \theta^{ij} \Pi^j(p) + T_i$ (where before, *i.e.*, in a private ownership economy without transfers, $T_i = 0$ for all i). Also as before, the associated allocation $(\mathbf{x}(\mathbf{p}^*), \mathbf{y}(\mathbf{p}^*)$ is called a WEA.

2.3 The First Welfare Theorem with Lump-sum Redistributions

Lump-sum transfers are also sometimes referred to as **non-distortionary** taxes. The main reason for this is that lump-sum redistributions don't "destroy" Pareto optimality (whereas other kinds of taxes usually do). Thus we have:

Theorem 2 (First Welfare Theorem, Private Ownership Economies with Lump-sum Redistributions) *Consider a private ownership economy with lump-sum redistribution* $(T_1, \ldots, T_I) \in \mathbb{R}^I$, $\sum_{i \in \mathscr{I}} T_i = 0$. *If each* u^i *is strictly increasing on* $\mathbb{R}^n_+$, *then every WEA is Pareto optimal.*

3 The Second Welfare Theorem with Production

In words, the second welfare theorem says that "whatever you want, you can have it (as long as it's Pareto optimal), all you need to do is make a suitable redistribution". The *you* in this sentence could be a planner or the government of a country.

Theorem 3 (Second Welfare Theorem with Production) *Consider a private ownership economy* $(u^i, e^i, \theta^{ij}, Y^j)_{i \in \mathscr{I}, j \in \mathscr{J}}$, *and assume that (i) each* u^i *satisfies assumption 5.1., (ii) each* Y^j *satisfies assumption 5.2., and (iii)* $\mathbf{y} + \sum_{i \in \mathscr{I}} \mathbf{e}^i \gg 0$ *for some aggregate production vector* $\mathbf{y}$.[1] *Let* $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$ *denote any Pareto optimal allocation. Then there exists a lump-sum redistribution* $(T_1, \ldots, T_I) \in \mathbb{R}^I$, $\sum_{i \in \mathscr{I}} T_i = 0$, *such that* $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$ *is a Walrasian Equilibrium Allocation for the private ownership economy with income transfers* $(T_1, \ldots, T_I)$.

I'll be talking much more about this result next week. We will not prove this result in class.

Please notice that the approach I've taken in this note is different from our textbook's. In GR, the second welfare theorem (Theorem 5.15) is written without spelling the redistribution concept out beforehand. It is, however, the same as ours, and I recommend that you spend a few minutes convincing yourself that this is so.

[1] See last week's lecture notes for the precise meaning of (iii).