

# Class 2: Assets and arbitrage

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These questions are related to the classical finance theory established by Modigliani and Miller. While we have addressed some of the concepts in the lectures, the actual problems will be novel.

1. a single firm is financed by a mix of equity and debt. Its future profits may be either good or bad. The payoffs to the firm's equity and debt are, respectively,

$$E^1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, D^1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}; \quad (1)$$

where the first row the payoff when the firm does well and the second when it does poorly.

- (a) does the firm go bankrupt in either state of nature?  
(b) an investor with a budget of  $B$  wishes to buy some quantity,  $q_E$ , of the firm's equity, and some quantity,  $q_D$ , of its debt in order to guarantee herself an income.<sup>1</sup> Solve for  $q_E$  and  $q_D$ ; explain your result.
2. A second firm is entirely equity financed. The payoff to its equity is

$$E^2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix};$$

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<sup>1</sup>In other words, she is looking for a  $q_E$  and a  $q_D$  such that she receives the same payoff whether the firm does well or poorly.

in the good and bad states of the economy, respectively.

- (a) under efficient markets, what is the relative prices of a share in firm 1,  $p_1$ , and one in firm 2,  $p_2$ ?
  - (b) the investor of question 1b now wishes to use her budget to ensure a constant income in either state, this time from a portfolio comprised only of the equity instruments. What quantities,  $q_E^1$  and  $q_E^2$ , should she buy of firm 1 and 2's shares, respectively? Explain your result.
3. Following [Myers \(2001\)](#), assume that a firm repurchases and retires all £1 mn of its equity, swapping this for a £1 mn perpetuity issue to its original share-holders. Further assume that it commits to never repurchase its new debt and that there is no credit risk.
- (a) Without taxes, what is the NPV of the firm's repurchase? Why?
  - (b) If the corporate income tax rate is 35%, and interest payments on debt are tax-deductible, what is the after-tax NPV of the firm's repurchase?
  - (c) Suppose, before the repurchase, the firm's share-holders faced a dividend income tax rate of 40% and a capital gains tax rate of 8%. Assume the firm paid no dividends, allowing residual income to accumulate as capital gains. On £100,000 of the firm's income:
    - i. How much tax did the firm owe?
    - ii. Roughly how much tax did its share-holders owe? When was it due? (Hint: share-holders are residual claimants.)
  - (d) After the repurchase, the firm pays an annual interest rate of 10% on its debt.
    - i. How much does it pay out annually in interest payments?
    - ii. How much does it save in corporate taxes?
    - iii. How much more do investors receive in interest income?
    - iv. How much less do investors receive in capital gains?
    - v. What is the net effect of the firm's repurchase on corporate, personal and capital gains taxes?

## References

- S. C. Myers. Capital structure. *Journal of Economic Perspective*, 15(2):81 – 102, Spring 2001.